Maths Is All Around You

Developing Mathematical Concepts in the Early Years

Marianne Knaus

TEACHING SOLUTIONS

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Most importantly, I would like to acknowledge the children and parents who gave permission to be included in this book. I thoroughly enjoyed sharing these mathematical experiences with you. I hope that learning mathematics will continue to be a joy to you all.

Foreword

Many years ago, I was lying on the grass at a preschool with a 4-year-old boy, Simon, beside me. We were looking at the clouds: white fluffy clouds in all sorts of shapes. We talked about the shapes we could see – elephants, towers, flowers, ships, crocodiles and so on. (We did not mention triangles, squares or rectangles.) Simon rolled over said, 'Bob, why do these clouds all have flat bottoms?' 'I don't know, ' I replied, 'but it's something worth finding out so let's explore it together.' We did explore the question and were both fascinated with what we found. I will not spoil it for anyone by disclosing our findings here except to say that since that day, I have looked at clouds quite differently.

Also some time ago, I entered a preschool room in which a young girl, Anthea, was 'swooshing' a ball above her head. I said, 'Can you throw the ball to me?' to which she replied, 'It's not a ball, it's a rocket ship and I am going to the moon.' Anthea ran off, avoiding any further conversation with me.

Both of these examples demonstrate the potential for mathematical learning in the early years of children's lives. They also illustrate that adults (even the same adult) can be both helpful and not so helpful in facilitating this learning.

In this book, Marianne Knaus has used her knowledge and expertise to ensure that the facilitation of young children's mathematics learning is helpful. She has used the unique Australian moment of national school and prior-to-school curricula (frameworks) to introduce a different way of thinking about the early years using the strands of the national mathematics curriculum. While this will raise some eyebrows, Marianne does it to promote successful transition in mathematics learning across the prior-to-school and school sectors.

The basis of the approach taken in this book is 'play'. This over-used, misunderstood and often maligned term is still central to early childhood pedagogical approaches and deserves its leading position in this book. As well, there are interesting explorations around the roles of educators, peers and families in young children's mathematics learning.

The bulk of the book consists of chapters on each of the strands of the Australian mathematics curriculum and how they might be experienced by children in prior-to-school settings. Clear links are made with *Belonging, Being and Becoming: The Early Years Learning Framework for Australia*. The chapters provide a great deal of detailed and practical advice about how young children's learning in these strands might be facilitated. Early childhood educators in both prior-to-school settings and the first years of school will find all of this very helpful.

The book is completed through an exploratory chapter on the use of information and communications technologies (ICTs) in young children's mathematics learning. Such a chapter can only be seen as a start to the investigation in the context of rapid development in the field. Marianne advocates a balanced approach to the implementation of ICTs in early childhood mathematics learning and sets the scene for further exploration by the reader.

I congratulate Marianne on her book which I can see will be very useful for practising early childhood educators in both prior-to-school and school settings, students preparing for such roles, and for families. Most of all, this book will benefit young children now and into the future. It will assist children like Anthea and Simon to explore their worlds and grow as powerful mathematicians. This, surely, is our aim.

Bob Perry, Professor of Mathematics Education, Charles Sturt University

About this book

The title of the book – 'Maths is all around you' – implies that in our daily experiences we encounter mathematics on a regular basis in one form or another, whether it be making breakfast (measuring), driving to work (spatial awareness) or buying a coffee (using money). We use maths in some way every day, and therefore it is an important life skill. Children are naturally curious and keen to learn about their world, including mathematics. This book is intended for educators and parents who would like to explore and investigate maths concepts to enrich children's experiences and extend their current thinking and learning.

For some people, maths is 'scary' and not something they feel confident about. Even though many educators and parents attempt to provide good mathematics experiences there is still a high level of anxiety about the teaching and learning of mathematics. These feelings can become obstacles to the provision of a rich maths program in early childhood.

Another problem that inhibits educators is the lack of awareness of what to teach in the early years and unfamiliarity of the particular concepts that can be introduced from an early age. This book presents a broad range of concepts and aims to widen the narrow view that maths for young children is just about numbers and shapes. The content includes pattern (early algebra), counting, number, early operations, measurement, shape and spatial awareness (geometry), matching, sorting, data analysis and the introduction of chance (statistics and probability).

The philosophy and pedagogy of the Early Years Learning Framework (EYLF) form the basis of this book. Incorporated throughout are examples of the EYLF principles, practices and learning outcomes that link to the various maths concepts. Mathematics and numeracy has become a high priority for curricula and is gaining much attention from governments and researchers. Unfortunately, some of this interest has lead to a misguided perception where children are introduced to formal mathematics too soon, demonstrating a 'top-down' approach of academic learning. This book recommends a balance of incidental spontaneous experiences combined with specific planned experiences incorporating mathematics concepts. Underpinning each chapter is a strong emphasis of a play-based approach. Maths can be integrated into routines and learning experiences throughout the curriculum and across a range of curriculum areas, including science, literacy, society and environment.

The aim of the book is to be a practical resource to enhance teaching practice. Everyone can teach maths – they just need to know how and what to teach. Being aware of the mathematics occurring in everyday contexts makes a

good starting point. As you read this book you will probably be surprised that many of the suggestions are things that you already do, but were unaware of the links to mathematics. Early maths concepts are embedded in language. Throughout the book the importance of language is emphasised with lists of vocabulary to use and questions to ask to provoke children's thinking. Also included are many practical ideas for experiences that can easily be implemented.

Each one of us can make a difference in the lives of children. I encourage you to introduce children to mathematics in an interesting and enjoyable way and to inspire the joy of mathematics, establishing a strong foundation for life.

Marianne Knaus

1 Teaching and learning mathematics

Young children are naturally curious and eager to learn about their surroundings and the world they live in. They begin learning about mathematics long before they start formal schooling (Perry & Dockett 2005). Young children construct their own understandings as they observe and participate in everyday life. The development of mathematical understanding occurs in a range of settings: at home, childcare and while shopping or at the park. Parents and educators, when they provide the necessary language, meaningful experiences and opportunities, can enhance children's early mathematical learning. But what are appropriate mathematics experiences and opportunities?

Mathematics for children under five years of age is often overlooked or misunderstood. Sarama and Clements (2009, p. 350) suggest that 'Teachers need to learn that appropriate mathematics for young children is wider and deeper than usually realized.' This book will hopefully lay aside some of these misconceptions and help to inform parents and educators how to actively engage children to make meaning and connection between mathematics and their everyday play experiences and exploration.

The garden at Roseworth Playgroup offers many opportunities for talking about mathematics concepts: the tomatoes can be counted as they are being picked, the growth of the plants can be compared and measured, the size and weight of the produce can be investigated as well as the shape and pattern of the leaves.

Attitude and philosophy

The starting point for educators is to firstly consider their attitudes and feelings about mathematics. These feelings are often subconsciously communicated to children and will impact on their learning. Beliefs about mathematics

can be influenced by previous experiences. Some people have developed very negative attitudes and have a low self-concept of their mathematical ability. This could be due to poor teaching practices they experienced, where emphasis was on rote learning and drills which took precedence over understanding and applying mathematics.

Being anxious about mathematics may lead to a feeling of apprehension about teaching it. Positive experiences with mathematics are often linked to educators who made it interesting, were helpful and engaged children. Attitudes and feelings can be transferred to our teaching. It is important that we foster a positive attitude towards mathematics and that it is an integral part of the curriculum for young children.

> Positive attitudes and competencies in literacy and numeracy are essential for children's successful learning. The foundations for these competencies are built in early childhood. (EYLF p. 38)

The National Quality Standard (NQS) requires that each service provides a copy of their current philosophy as part of the quality improvement plan (QIP). The philosophy statement needs to address the requirements set out in the NQS which include the beliefs and theoretical values of all staff members. Developing a philosophy requires reflection and consideration of values and attitudes. This may be the time for staff to think about their philosophy and attitude toward teaching mathematics. Unfortunately, mathematics in early childhood settings quite often has a low priority, especially compared to literacy.

Theories of mathematics

Piaget, Vygotsky, Bruner and Dienes have all made important contributions to the teaching and learning of mathematics. All are cognitive psychologists, and their views are based on constructivism. The theory of constructivism suggests that knowledge is not passively received, but actively constructed by the learner. Piaget (1967) believed that children acquire knowledge by constructing ideas as they interact with the environment. Mathematics is an active process – not just through physical manipulation with concrete materials but through acting on prior knowledge to construct new meaning.

Vygotsky (1934), unlike Piaget, emphasised that learning was a social process and that children learn from more experienced peers and adults through shared experiences. Bruner (1972) believes that play is a medium for socialisation and that the educator, through quality interactions, acts as a scaffold for children's learning. Dienes' (1967) emphasis was on materials, and that the early stages of mathematics learning centred on free play and the investigation and exploration of these materials, before moving to more abstract thought processes.

> Different theories about early childhood inform approaches to children's learning and development. (EYLF p. 11)

The role of the adult

There are many opportunities to enrich mathematical understanding throughout the day. As children play and explore their environment, educators can take advantage of real-life opportunities to incorporate learning. It is necessary to be available and ready to use the teachable moment. For example, while children are negotiating an obstacle course we can incorporate spatial knowledge by using positional language – *'You've climbed up to the top of ladder. Show me how you can go down. Can you go around the tree? What about underneath the blanket? You've gone around the whole course.'*

As children arrive in the mornings we can use the language of ordinal number by suggesting that they were the first or second child to arrive that morning. Or while eating lunch introduce shape by talking about how the sandwiches were cut into triangles today. It is in these spontaneous everyday opportunities that we can make meaning and connections about mathematical understandings.

Educator Judy Warren helps Lachlan and Nathan with a picture activity book to position matching magnetic clothing parts to the appropriate area of the body. Being available and talking with the children allows for mathematical conversations to take place.

High-quality learning requires a balance of planned experiences and informal spontaneous experiences that occur during the course of the day. Educators are flexible in their roles as they interchange between 'child led, child initiated and educator supported learning' (EYLF p. 15). It is important for educators to enjoy what they are doing and to model an appreciation of mathematics and learning.

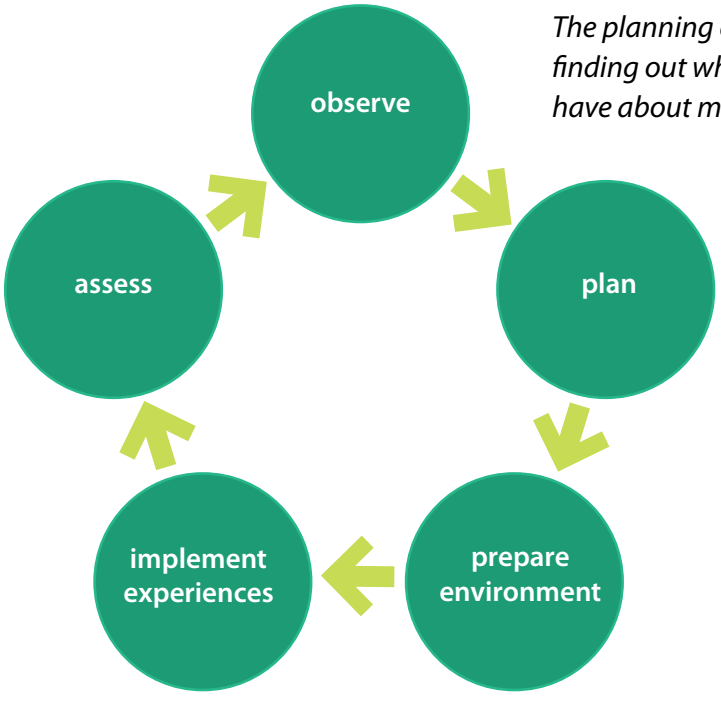
The quality of interactions with educators has a direct impact on the child's learning of mathematics. It is important to listen, watch and observe and to be available as a source of information to enrich understanding. An effective teacher of mathematics will ask questions to provoke children's thinking and introduce the language of mathematics to help children see the connections

between the world and mathematical concepts. The learning becomes purposeful and is actively promoted by the educator. This is referred to as intentional teaching in the Early Years Learning Framework.

> They use strategies such as modelling and demonstrating, open questioning, speculating, explaining, engaging in shared thinking and problem solving to extend children's thinking and learning. (EYLF p. 15)

It is important to specifically plan for mathematics learning, and not just rely on incidental opportunities to occur. Planning mathematical experiences firstly requires an understanding of what children already know.

Children learn at different rates and bring existing knowledge from their social and cultural context. Finding out about their prior knowledge happens through observations and records of children's involvement in play – their strengths and interests, and what resources they use and how they use them. We can then match this information to learning outcomes based on mathematical content knowledge and combined with teaching strategies to plan appropriate experiences. Mathematical content knowledge is detailed in later chapters.



The planning cycle begins with observing – finding out what knowledge children already have about mathematical concepts.

Teaching mathematics through play experiences

Play is used as a pedagogical tool for the implementation of learning outcomes, and it is through play that children acquire basic mathematical understanding. A range of play experiences can be provided, for example: sensory play, block play, construction play, dramatic play, outdoor play, music and movement, cognitive games, and creative play. Tucker (2010) recommends that during play experiences, quality interactions between educators and children are necessary to enhance and support mathematical development.

> Educators are also responsive to children's ideas and play, which form an important basis for curriculum decision-making. In response to children's evolving ideas and interests, educators assess, anticipate and extend children's learning via open-ended questioning, providing feedback, challenging their thinking and guiding their learning. They make use of spontaneous 'teachable moments' to scaffold children's learning. (EYLF p. 15)

What is important is for the educator to make meaning and connection to mathematics within the experience. In block play for instance, opportunities for learning about measurement, shape and space and number can be incorporated. Dramatic play can include measurement, shape and number knowledge. Well-planned play experiences build the conceptual links between many of the mathematical ideas and the real world. For example, going shopping requires knowledge about number and money; children can play out these real-life experiences and adapt this to new learning.

Through play, children are able to engage in higher order mental processes as the constraints of the real world do not exist. Educators help children to move to more mature levels of play through scaffolding and questioning (Bodrova & Leong 2007). Children also need to revisit play experiences they have been involved in. It is through repetition that they are able to practise the new skills acquired and to reinforce the mathematical concepts they are learning. As children re-examine materials and experiences, they may use them differently as their understanding increases.

Playing in the sandpit provides Thuy with the opportunity to learn about measurement. The language to use here to support the concept includes terms such as heavy and light, full and empty, and making comparisons between them.

Numeracy and mathematics have a much higher profile in education today than ever before. However, the importance of learning mathematics knowledge early is often confused with formal learning too early. This has resulted in a push-down effect of academic learning in early childhood. The introduction of worksheets is one example. Using worksheets does not encourage active hands-on learning or the ability to make connections between the real world and mathematical understanding. Formal mathematics experiences too early can turn children off mathematics. That is why a play-based approach is essential.

2 The foundations of mathematics

Everyday concepts

We use mathematics every day in some way. Think about the many ways you refer to mathematical knowledge: telling the time, buying a coffee, setting the table, driving to work, checking the weather, making a cake.

Signs and symbols are commonplace in the everyday environment. Mathematics is a part our world each day but is often taken for granted.

Knowledge of mathematics is an essential life skill – a tool for living. These everyday practices are an important part of children's lives, and according to Fleer and Raban (2007, p. 17)

> Children in their home and community have developed a series of everyday concepts that help them make sense of their everyday lives. These everyday concepts are part of their everyday practice.

Daily, children encounter mathematical experiences that require the meaningful use of numbers, measurement, shape and spatial thinking. They are constructing a great deal of informal knowledge based on their experiences. Copley (2001, p. 6) states that 'these constructed ideas are unique to each child and vary greatly among children the same age'.

Children's understanding of mathematics can be like a puzzle. They are constantly making connections over time, putting the pieces in place. The educator's role is to observe to find out what it is that they don't understand and try to help put the pieces together. In this way, everyday concepts can be transformed into abstract concepts (Fleer & Raban 2007).

To support the development of abstract thinking, the educator needs to understand mathematical concepts and recognise the potential of everyday events to help provide the links for learning. To think abstractly is when mental thought replaces the action. The path to concept development moves from informal understanding to formal knowledge.

> Young children move from concrete, to representational, and then to abstract thinking. Children initially discover relationships between materials and events in the three-dimensional world. Concrete materials serve as the central tool for children to think about objects, actions and events. Pictures and symbols take the place of concrete objects only after children have built a foundation through sufficient direct experience. Over time, mental images emerge for children to use as a reference for direct experience. (Schwartz 2005, p. 53)

Play and participation in everyday life can provide the context for mathematical understanding. It is important to use everyday experiences as a starting point to embed mathematical concepts. Children discover best through personal meaningful opportunities when the learning is grounded in first-hand experiences.

Gumboots lined up outside a room at Unicare Early Childhood Centre. There are many different sizes and patterns which are matched in pairs. These visual representations help children make meaning and connection to everyday life experiences.

> "It is essential that the mathematical ideas with which young children interact are relevant and meaningful in the context of their current lives. (EYLF p. 38)

Everyday experiences are informal and occur naturally in the course of the day. Examples for spatial awareness might include painting, referring to how they have covered the whole area, or up here and down there. When setting the table in preparation for lunch, comment that the spoons go next to the plate. Folding the washing can incorporate number as items are grouped together: all the washers are in this pile and the smocks over here. When cutting the fruit, talk about halves and quarters, introducing fractions. What is significant is the use of the teachable moment to make connections to the mathematical concepts.

4 Early number experiences

What is numeracy?

Many people assume that numeracy only encompasses number learning. However, numeracy involves being able to apply numerical concepts to all aspects of mathematics and life in general. To be numerate requires children to have a sound understanding of mathematics and be able to apply numeric skills to the demands of everyday life.

> “Numeracy is the capacity, confidence and disposition to use mathematics in daily life. (EYLF p. 38)

Numeracy involves a good understanding of number sense and the processes involved to make meaning of the world. The Australian Curriculum (ACARA 2010) has reported that numeracy is having the skills and knowledge to use mathematics across all learning areas.

Number sense is applied to other mathematics strands such as measurement, geometry, probability and statistics. For example, in geometry, a triangle has three sides as opposed to a rectangle that has four sides. To be able measure effectively, knowledge of number is required to know accurately how long or heavy something is.

> “Spatial sense, structure and pattern, number, measurement, data augmentation, connections and exploring the world mathematically are the powerful mathematical ideas children need to be numerate. (EYLF p. 38)

From a young age, children are exposed to seeing number symbols in their environment. It is helpful if educators say the numbers, sing number songs or rhymes and even clap beats to the numbers.

Knowledge of number and the skills and processes involved is required to be able to use mathematics successfully in a wide range of contexts. Therefore, for children to develop the dispositions to use mathematics in everyday life and to be able to apply it effectively, the teaching and learning needs to be meaningful, with real-world experiences. The foundations of numeracy begin in infancy, and more complex abstract concepts develop with experience and time.

Why is number learning important?

Numbers are used every day, and for children to learn number sense they require a range of experiences to successfully be able to transfer their skills from one context to another. According to Sarama & Clements (2009), number and developing number sense is the most important part of mathematics learning. Number sense involves making the connection between quantities and counting (Charlesworth 2012).

Research suggests that targeting number sense early prepares and equips children to learn more complex concepts later on (McGuire, Kinzie & Berch 2011). The infant, toddler and preschool years offer many opportunities to capitalise on children's informal number sense knowledge. Research studies have found that number sense skills developed in the early years are predictive of mathematics achievement in the primary and secondary years of schooling (Sarama & Clements 2009).

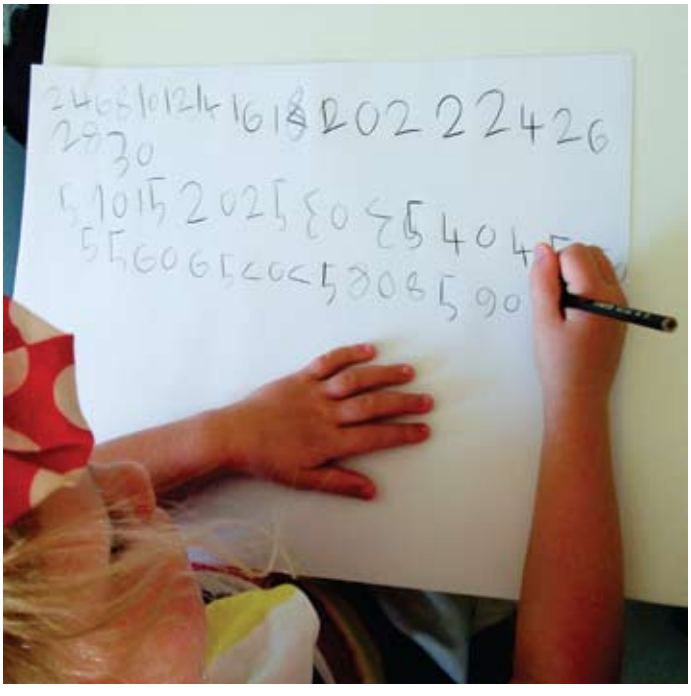
Ryland was catching fish that were numbered. He grouped the fish into sets of colours and then counted them. Ask questions such as 'How many red ones are there?' 'Which has more, the red ones or the green ones?' 'How many more?'

What to teach

Firstly, it is important to get to know individual children's prior knowledge. All children have different life experiences with regard to number knowledge. Therefore, educators need to find out what they do know to be able to move them from their present constructions to abstract concepts in order to optimise their learning.

Learning Outcome 5: Children are effective communicators.

> Children begin to understand how symbols and pattern systems work. Educators promote this learning, for example, when they:
>
> - engage children in discussions about symbol systems, for example, letters, numbers. time, money and musical notation
> - encourage children to develop their own symbol systems and provide them with opportunities to explore culturally constructed symbol systems (EYLF p. 43)



Hannah wrote numerals to explain how to count in twos up to 30. She then wanted to show us how she knew to count in fives up to 100.

Planning experiences

There are numerous situations during the day where children practise counting and learn about number. It is important to capitalise on the teachable moment. During the infant and toddler years, most of this learning will be in informal situations.

For preschoolers, provide a balance of informal everyday opportunities and focused experiences. Number sense develops over a long period of time, so long-term planning and short-term planning are both important for continuity and to help children progress and extend their concept knowledge in a sequential manner. Children need a wide range of varied experiences to learn about number.

- Provide opportunities to recite number names in order and to recognise numerals (number songs and rhymes, books, games, number frieze, displays of objects to count).
- Include the language of quantity to make comparisons (more, same, not many, lots).
- Provide visuals – number charts, telephone books, menus, catalogues.

- Environments and learning centres can be set up to include experiences involving number concepts (while playing with dough, blocks, construction, role play).
- At group times and transitions, include books, rhymes, songs and games about number.
- Use dice, dominoes, matching games, manipulatives and board games.

Rhonda, an educator, sings the song 'Five Little Ducks' while acting it out with hand and finger puppets. Children are introduced to notions of take away and adding through such simple experience as these.

When assessing children's number knowledge, take regular observations to note what individuals can do and what they know. Don't assume – listen and watch carefully for errors and be aware of their use of language. Use closed questions (*How many?*) and open questions (*How do you know?*). It is important to know what you are looking for, so use assessment key points. Montague-Smith (2009) suggests things to look for when observing counting and number skills:

- Knows number names in order to ...
- Counts by pointing
- Counts by touching and partitioning
- Knows that the last number in the count is the cardinal value of the set
- Subitises for small quantities
- Recognises that a quantity is larger/smaller than another
- Combines two small sets and says how many
- Takes away a small quantity and says how much is left
- Makes fair shares
- Uses appropriate mathematical language for addition, subtraction and division
- Begins to recognise and name numeral

Language to use

- more, less, same, fewer, many, group, count
- counting numbers: zero, one, two, three ...
- ordinal numbers: first, second, third ...
- add, take away, share, equal
- part, whole, pieces, some, half, quarter

Questions to ask

- How many are there?
- How many did you count?
- Have you counted them all?
- Who has more/fewer?
- How many more do you need?
- What if you need one more, how many then?
- What if you started counting with ... are they the same number?
- What number comes after ...? What number comes before ...?
- How many more are in this group?
- What if you put them into a different group, how many then?
- If I add one more, how many are there? If I take one away, how many are there?
- What if I cover some of these, how many are hidden?
- Share these between us, how many do we have?
- What number is that? (Referring to numerals)
- Can you find the number card to show how many there are?
- Can you put the number names in order?

Recycled materials – pegs and ice-cream containers – were used to develop this number experience. The skills involved for Toby included recognising numerals, placing the numbers in order from 1 to 5, knowing the quantity of each number and matching the quantity of pegs to the correct numeral.

Number experiences

Counting

- Provide interesting displays for children to count. Items such as a row of elephants lined up on a shelf, boxes and baskets of interesting objects, natural materials such as stones, seed pods and pine cones, picture friezes on the walls.
- Incorporate number rhymes, songs, poems, puppets and books spontaneously throughout the day.
- As part of daily routines, count how many children, cups, paint brushes.
- Have a container that the children can drop counters into and hear the sound as each one drops. Count aloud with the children.
- Add manipulatives to learning centres, for example, shells and figurines to the block area to encourage counting in play.

Using counters used to represent the number quantity.

- For preschoolers use five frames or ten frames to use with counters to tag, represent numeric quantities, explore combinations of numbers and partition numbers. Children could roll a dice and represent the number on a five frame or ten frame.

Cooking experiences

- Incorporate counting – how many cups or teaspoons?
- Provide a laminated recipe to the encourage recognition of symbols on the recipe.
- Use language of division and fractions as you halve the mixture, divide into patty papers, use half a tablespoon, or a quarter of a cup. How many can each person have if we make 12 biscuits?

Games

- There are a variety of card games, Uno, dominoes, number snap, lotto, board games (Snakes and Ladders), concentration and matching games that can be constructed or bought commercially. These help children to recognise numerals, count and match numerals.